A Simple Logarithmic Receiver*

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Summary—A receiver with a logarithmic input-output law can be used with advantage on radar equipment, but it has hitherto been difficult to obtain such a law without the addition of a considerable number of tubes. One system by which such a law can be obtained is described and a circuit is given which shows how the method may be simplified to produce a receiver which can be either logarithmic or linear by the operation of a single switch. A method of predicting the shape of the input-output curve of such a receiver is described. The measured input-output curves for three logarithmic receivers of this simplified type are shown and the relevant circuit values specified. Some design notes are added.

A note on the use of the logarithmic receiver as an anticlutter device is followed by consideration of the display of signals from a logarithmic receiver, and a brief discussion on bandwidth variations.

I. Introduction

A RECEIVER which has a logarithmic relationship between output volts and input volts over its working range of inputs may be called, perhaps loosely, a logarithmic receiver. Receivers of this type can be used with advantage in place of linear receivers in radar equipments.

For instance:

(a) A logarithmic receiver preserves amplitude discrimination at its output between all signals fed to its input, throughout the working range of inputs (usually about 80 db). For this reason no gain control is necessary on logarithmic receivers, and the receiver output can be calibrated directly in decibels, the calibration not being dependent upon some arbitrary setting of gain control.

(b) A radar equipment being used over sea will receive random returns from the sea surface (sea clutter). Such signals produce, at the output of the receiver, a waveform to some extent similar to random tube and circuit noise in which the peak value of the impulses bears a constant ratio k to the mean value. Returns from rain clouds bear an even closer resemblance to the receiver random noise structure as displayed on a cathode-ray tube. If, therefore, noise plus sea clutter or noise plus rain clutter is fed to a logarithmic receiver which is followed by a simple RC differentiating circuit to remove the mean value, the peak output level of noise plus clutter is reduced to a constant value across the cathode-ray tube trace.[1] Since the receiver cannot saturate, signals which have an amplitude greater than the peak clutter amplitude, then appear quite clearly above this constant level of noise plus clutter on the display. The logarithmic receiver may thus replace the linear receiver fitted with a sensitivity time control (STC) with the advantage that no adjustment is needed as with the STC, to allow for differing degrees of roughness of the sea.

There are several methods by which a receiver may be made to give a logarithmic relationship between input and output.[1,2] The author's work has been devoted entirely to the successive-detection method.

II. The Successive-Detection Logarithmic Amplifier

An ordinary linear intermediate-frequency (IF) amplifier has a chain of amplifying tubes the final one of which feeds a detector. Over a small range of input the output bears a linear relation to the input, but linearity rapidly ceases to hold and very soon an input is reached beyond which no further increase in output is possible, the final IF tube being saturated.

The successive-detection amplifier differs from this in that it has a detector after every IF stage so that each stage besides feeding an IF input to the next stage in the normal way, can make an independent contribution to the video output of the amplifier, the output of all the detectors being combined. The input to such an amplifier can clearly be increased until every IF amplifying tube in the chain is saturated before the total output reaches a saturation value. This means that if we have two amplifiers, each of n stages with stage gain m, the one linear and the other of the successive-detection type, the latter amplifier can accept an input m^{n-1} times as great as can the linear amplifier before saturation takes place.

The following simple consideration of the successive-detection amplifier with n stages, of gain m, in which the output of the detectors are all added directly, will show that the input-output relation is approximately logarithmic.

Let us assume the saturation output from each of the IF stages to yield a rectified voltage V from the detector attached to the stage. We can imagine a signal generator to be connected to the input of the amplifier and its attenuator adjusted until the saturation output V is just derived from the last detector at, say, a signal generator input of v. If the generator input is now turned up m times to mv, we shall get an additional output V derived from the penultimate $(n-1)$ IF stage which now saturates giving a total output $2V$ from the last two detectors. If the generator output is now turned up a further m times to m^2v, we get an additional output V from the $(n-2)$th IF stage giving $3V$ in all. It is clear that this step-by-step process can be continued until the first IF tube in the successive-detection

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[1] R. V. Alred and A. Reiss, "An anti-clutter radar receiver," *Jour. IEE* (London), vol. 95, part 3; November, 1948.

[2] S. N. Van Voorhis, "Microwave Receivers," *Rad. Lab.*, vol. 23, pp. 583–606, McGraw-Hill Book Co., Inc., New York, N. Y.; 1948.

amplifier saturates giving a total output from the amplifier of nV for a signal generator input $m^{n-1}v$. The signal generator inputs for all these steps will be in geometric progression and the amplifier outputs in arithmetic progression, hence an approximate logarithmic input-output law has been achieved.

Approximate because in the simple argument above the input steps taken are from the condition for saturation of one IF tube to saturation of its predecessor in the chain, and clearly each of these points lies on a perfect logarithmic curve. Whether or not intermediate points are on the curve will depend upon the law connecting input and output voltage for any single typical IF stage, up to the input which saturates the stage. This point is given further consideration in Section V.

III. The Original Logarithmic Amplifier Circuit

The author's first experiments were with a 10-stage amplifier having a bandwidth of 8 mc centered at 30 mc. The amplifier incorporated band-pass coupled circuits of conventional type. From each control grid a detector (diode), fed video signals to a load resistance placed between grid and ground of a buffer video amplifying tube. Each stage in the logarithmic amplifier, therefore, required three tubes (one IF amplifier, one detector, one video amplifier). The outputs of all the video amplifiers were combined by giving them a common anode load. The anodes were connected together through an artificial delay line having one section between each anode, the delay per section being equal to the delay per stage of the IF amplifier. The delay line was terminated at each end by its characteristic impedance, the common anode load of the video amplifiers being formed by the parallel value of the two terminating resistances.

The video delay line is necessary in all amplifiers of this type when used for pulse reception, because the output pulse is built up from elements derived from each IF stage in the amplifier each element being therefore subject to a different IF delay. The delay line by bringing all the elements together at one time at the output, prevents the output pulse from having a stepped leading edge.

This type of amplifier is, of course, too costly in tubes and space to be of general application, and the author has derived a much simpler version. Before passing to this, however, some of the results derived from the above type of amplifier will be of interest.

Two exactly similar 10-stage amplifiers were constructed, since it was desired to divide the signal in one channel by the signal in another by subtracting the amplifier outputs. Miniature pentodes were used for the IF and video amplifying tubes and diodes as the detectors. No selection of tubes was necessary to match the amplifier characteristics. Fig. 1 shows the input-output curve for the amplifiers, the readings for one amplifier being represented by dots and those for the other by crosses. Fig. 2 shows the curves repeated after a complete change of tubes in the amplifier, and without any preselection of tubes.

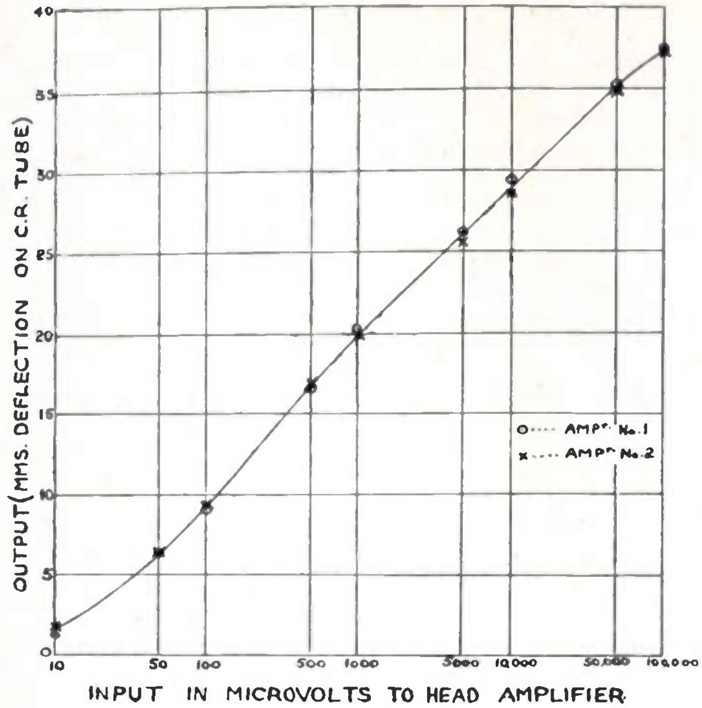

Fig. 1—Input-output curve for matched pair of receivers with log amplifiers of type using video buffer tubes.

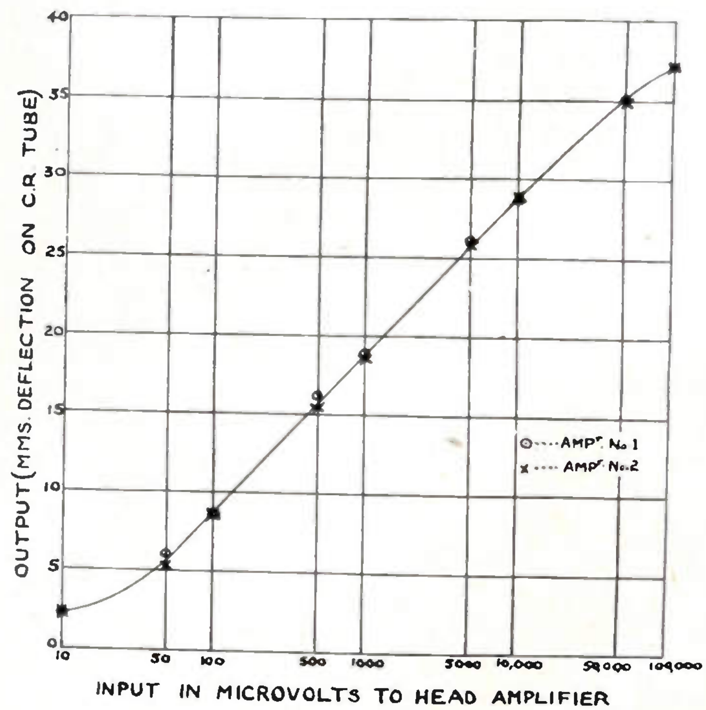

Fig. 2—Curve of Fig. 1 repeated after complete change of tubes, and with no attempt at tube selection.

IV. The Simplified Logarithmic Amplifier Circuit

Fig. 3 shows the circuit of the simplified amplifier. To save space, three stages only are shown. The diode detectors have been replaced by germanium crystals and the chain of buffer video amplifying tubes has been

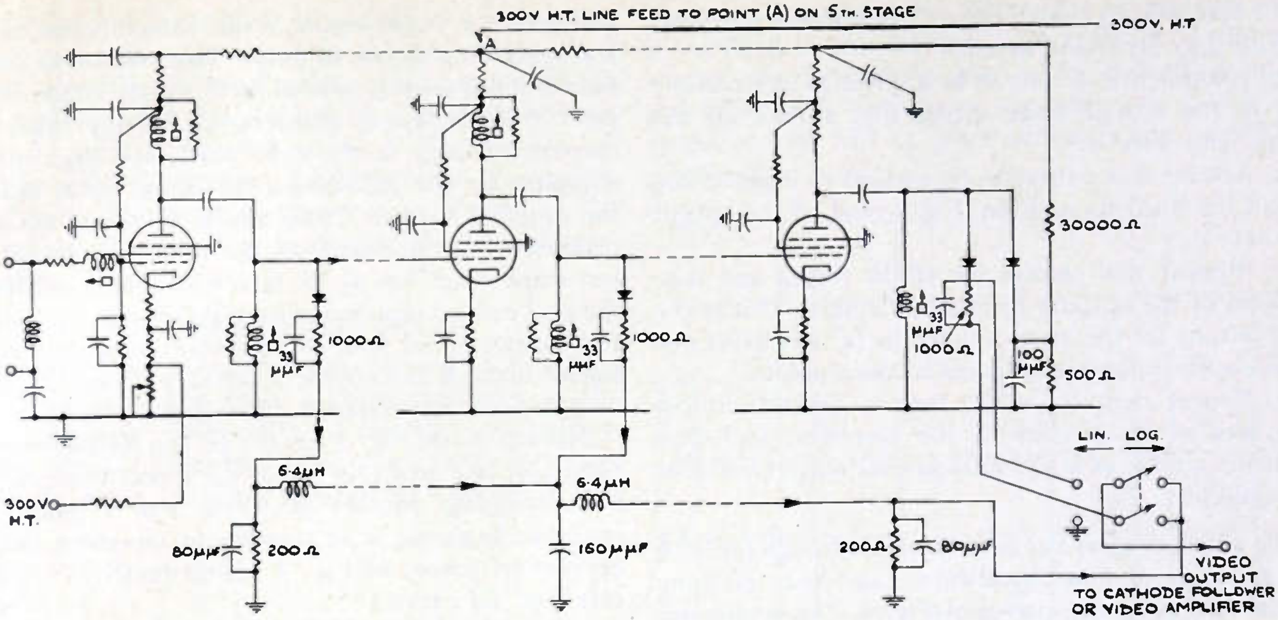

Fig. 3—Circuit of first and tenth stages of successive-detection logarithmic amplifier together with a typical intervening state. IF stages band-pass "top-end" capacity-coupled.

cut out. Isolation of the detectors from each other has been achieved sufficiently well by giving all the crystal detectors an individual load, and combining the load currents in a small common section formed by the terminating impedances of the delay line. The biasing effect of one detector upon its predecessor is now tapped down by the ratio of individual load to common load and can be made small enough not to matter. The purpose of the extra crystal shunting the last crystal detector will be explained in the next section.

A useful feature of this type of amplifier is that it can be switched to be an ordinary linear amplifier by a simple two-pole two-way switch as shown in Fig. 3. In the linear position the delay line is left isolated (but still terminated), and the amplifier output taken from the upper end of the last detector load which has its lower end switched to ground instead of to the delay line. In this condition, the noise output from the amplifier will, of course, be much greater since it is no longer tapped down by a potentiometer network, but a preset gain control can be included in one of the earlier stages to readjust the noise level to the working value.

Experiments were performed on three amplifiers of this type: (a) a 10-stage amplifier of 8-mc bandwidth centered at 45 mc; approximate gain, 8 db per stage; (b) a 5-stage amplifier of 4-mc bandwidth centered at 60.5 mc; approximate gain, 18 db per stage; and (c) a 4-stage amplifier of 0.5-mc bandwidth centered at 13.7 mc; approximate gain, 20 db per stage.

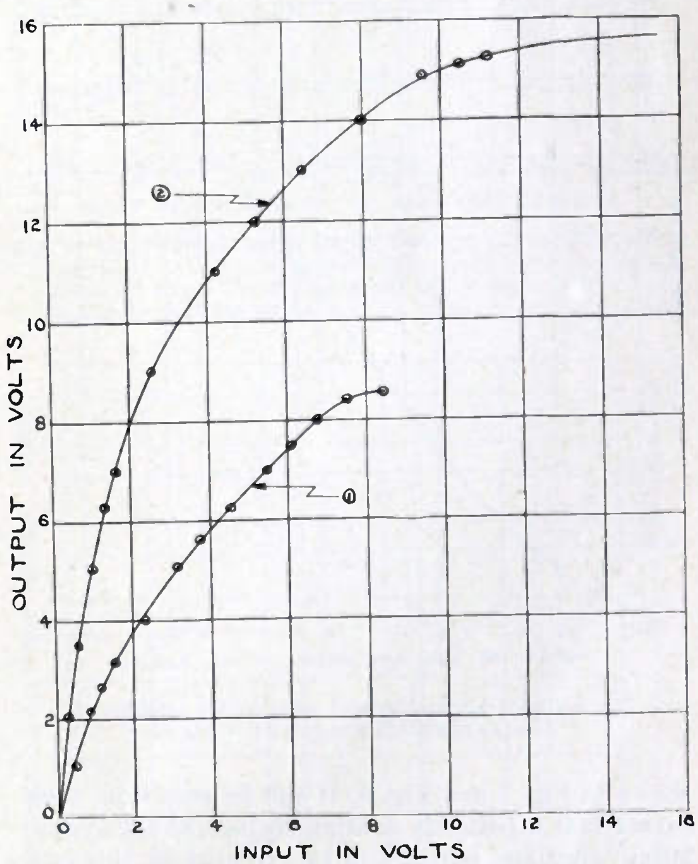

Fig. 4—Input-output curves for typical single IF stages of log amplifiers of 8-mc bandwidth (curve 1) and 4-mc bandwidth (curve 2).

V. The Input-Output Relationship

Fig. 4 shows the curves relating input and output for a typical single IF stage of the amplifier of 8-mc bandwidth, and the amplifier of 4-mc bandwidth. Using these curves it is possible to construct input-output curves for the complete IF amplifiers in the following way. Taking the case of the 10-stage amplifier:

(a) Assume a small input to be applied to the grid

of the first tube in the amplifier. From Fig. 4 (curve 1) read off the output and note it.

(b) Assume this output to be applied as input to the grid of the second stage. From Fig. 4 read off the output and note it.

(c) Assume this output to be applied as input to the grid of the third stage. From Fig. 4 read off the output and note it.

(d) Repeat this process for all 10 stages and take the sum of the outputs from all 10 stages. This is the total output for the input applied in (a) and gives one point on the input-output curve of the amplifier.

(e) Repeat steps (a) to (d) for any desired number of values of input volts to the amplifier, and thus construct a table of input volts against output volts for the amplifier.

The above process has been worked through for both the amplifier of 8-mc bandwidth, and that of 4-mc bandwidth, using the curves of Fig. 4. The results are

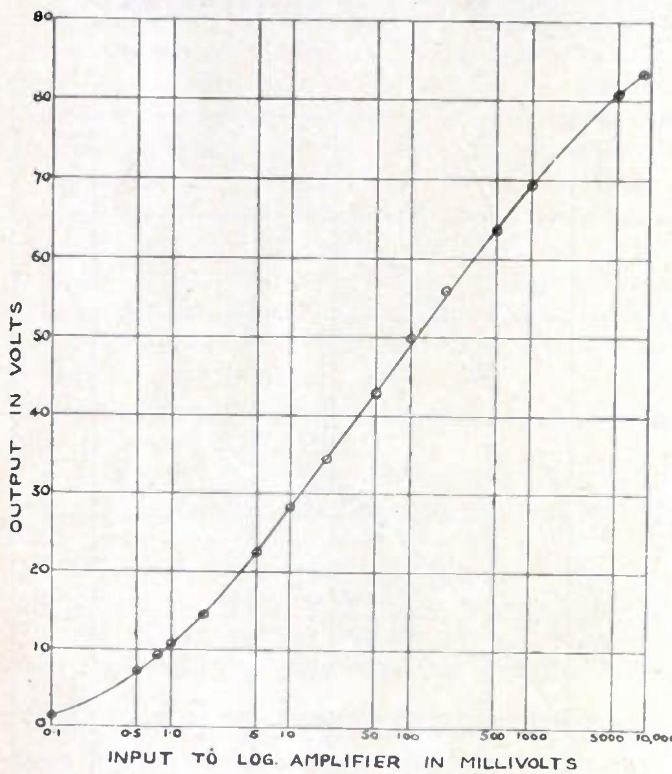

Fig. 5—Predicted input-output curve for log amplifier of 8-mc bandwidth, using curve 1 of Fig. 4.

shown in Fig. 5 and Fig. 6. It will be seen from these curves that a perfectly continuous logarithmic law relating input and output can be expected for this type of amplifier, and this disposes of the point left unanswered in Section II.

It is essential that all stages in the amplifier should be similar not only with regard to tubes but to operating conditions, within the normal limits allowed by component tolerances.

The curves of Fig. 4 show that the IF stages are amplifying linearly for small inputs, the stage gain falling off for larger inputs, until the saturation output is reached with a gain of unity. This saturation output level is influenced by several factors, such as the curvature of the dynamic characteristic of the tube, grid current damping in the tube itself, and grid current damping by the following tube acting back through the coupling circuits. These effects will be of the same magnitude for all stages in the amplifier, except the last stage which has no IF tube following it to provide the grid current damping effect. To prevent the saturation output of the final IF tube from being as a result higher than all the others, a suitably biased crystal is used to limit the output to the required level (Fig. 3).

The curves of Fig. 4 cannot be conveniently represented by one equation except by a polynominal with a considerable number of terms. For a qualitative analysis, however, it is possible to represent the experimental curves of Fig. 4 approximately by parabolas, e.g., for curve 1:

$$(v_{n+1} - 8.43) = - 0.111(v_n - 8.73)^2. \tag{1}$$

Assuming a parabolic law, the output voltage of the amplifier which is given by the sum

$$u = \sum_{1}^{n+1} v_n$$

can be expressed analytically as a function of input voltage v_0. This has been done and the result shows u to be a linear function of log v_0 over a restricted range of v_0, whereas experimentally this holds over a considerably wider range. The agreement, therefore, is of a qualita-

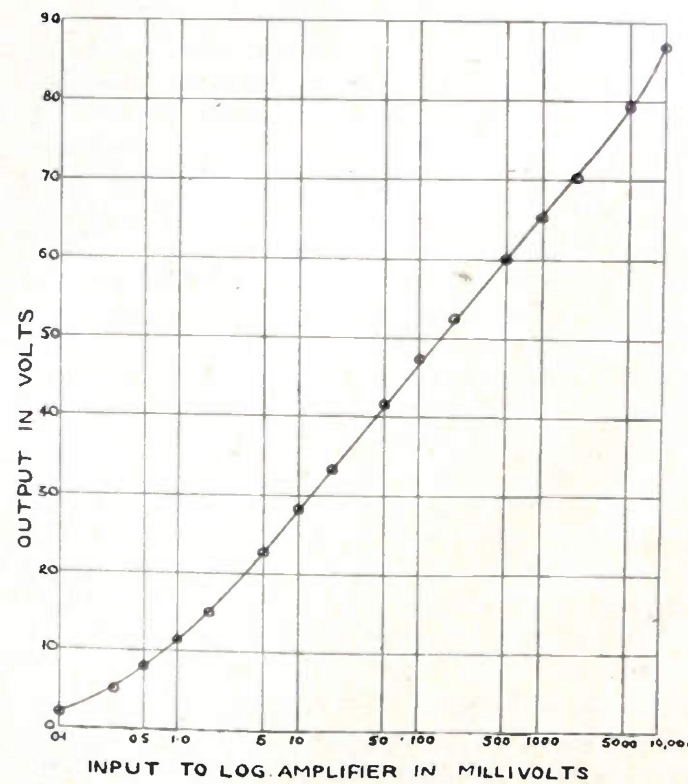

Fig. 6—Predicted input-output curve for log amplifier of 4-mc bandwidth, using curve 2 of Fig. 4.

tive nature only, as is to be expected since a parabola is only a crude approximation to the experimental curves, especially for small values of v_n. Attempts have been made to solve this problem by finding expressions which fit the experimental curves of Fig. 4 more accurately, but the algebra rapidly became intractable. The graphical solution is, therefore, submitted as the only practical method of predicting the over-all performance.

VI. Experimental Results

The experimental results were obtained by feeding a 10-microsecond pulse from a signal generator to the head amplifier of the logarithmic receivers and measuring the height of the output pulse on a cathode-ray tube. Fig. 7 (curve 1) is an input-output curve for the

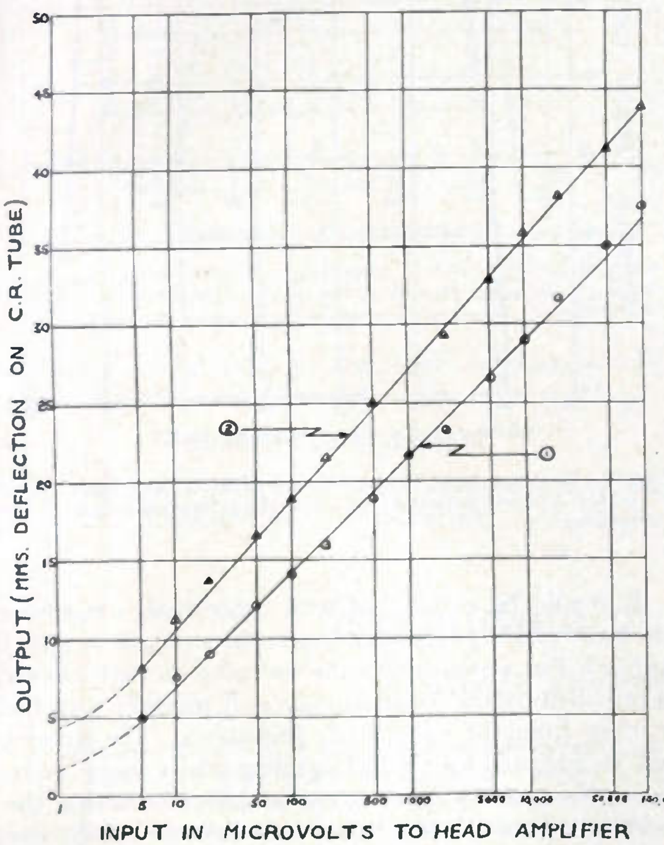

Fig. 7—Input-output curves for log receiver of 8-mc bandwidth. Individual detector loads of log amplifier = 1,000 ohms. Curve 1, common detector load 100 ohms, detectors on all stages. Curve 2, common detector load 250 ohms, detectors on alternate stages.

amplifier of 8-mc bandwidth, having the detector circuit values of Fig. 3. The law is closely logarithmic over an input range of 86 db. Curve 2 is taken for the same amplifier using a common load of 250 ohms in place of the 100 ohms, and with crystal detectors applied to alternate stages only. It is, therefore, possible to obtain a smooth logarithmic response from a multistage amplifier by using crystals on every other stage.

Fig. 8 shows the input-output curves for the amplifier of 4-mc bandwidth. Curve 1 was obtained with crystals on every stage, and curve 2 with crystals on alternate stages. The individual loads were 5,600 ohms, and the common load, 340 ohms. Although a satisfactory result is obtained in this instance with crystals on alternate stages, the practice is not to be recommended on amplifiers with 5 stages or less, since the failure of a single crystal will be more serious when so few are employed.

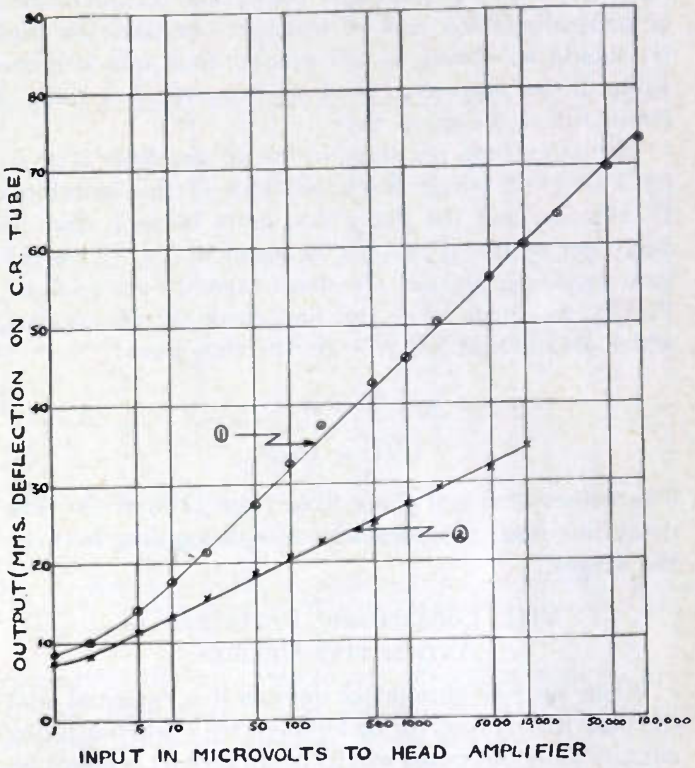

Fig. 8—Input-output curves for log receiver of 4-mc bandwidth. Individual of log amplifiers = 5,600 ohms. Common detector load = 340 ohms. Curve 1, detectors on all stages. Curve 2, detectors on alternate stages.

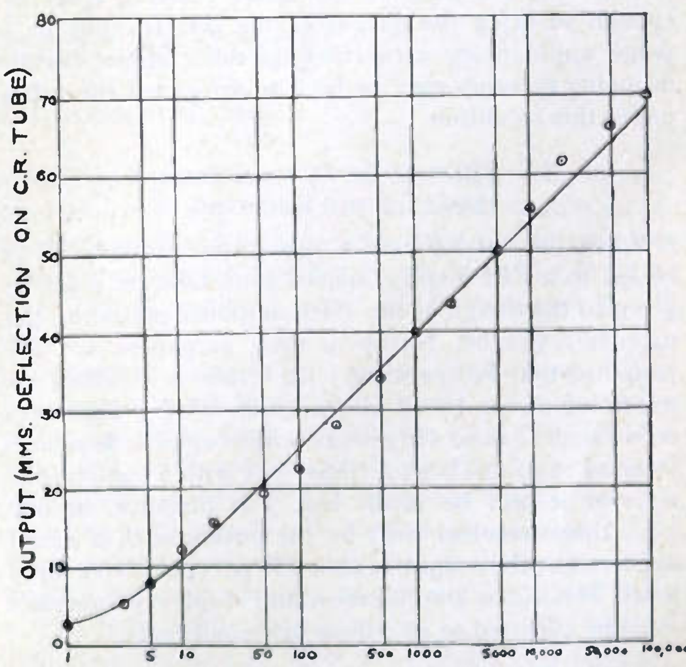

Fig. 9—Input-output curve for log receiver of 0.5-mc bandwidth. Individual detector loads 15,000 ohms. Common detector load 3,000 ohms.

Fig. 9 shows the input-output curve for the amplifier of 0.5-mc bandwidth. The individual loads were in this case 15,000 ohms and the common load 3,000 ohms.

VII. Design Notes

The value decided upon for the crystal individual loads (R) will be governed by the normal considerations of pulse distortion and IF loading. The common load (r) should be chosen as not greater than one-fifth the value of the individual load, and one-tenth is quite a reasonable fraction to take.

The delay time per stage in the IF amplifier is given by $T_D = 1/\pi B$ where B = bandwidth of the individual IF circuits, and the delay line must be such that its delay per section $T_s = \sqrt{LC}$ is equal to T_D. (L = series inductance per section, C = shunt capacity per section.) Finally we must have the line correctly terminated, which means that $\sqrt{L/C} = 2r$. We thus have:

$$\sqrt{L/C} = 2r. \tag{2}$$

$$\sqrt{LC} = 1/\pi B. \tag{3}$$

The values of L and C are fixed from (2) and (3). The delay line itself provides adequate decoupling between the stages.

VIII. Logarithmic Receivers as Anticlutter Devices

When used as anticlutter devices it is essential that the logarithmic receiver be followed by a differentiating circuit.[1] Only one point will be made here. It is essential, when used as an anticlutter measure, for the receiver to be logarithmic not only up to the maximum signal level received, but also down to below noise level. This means that the gain of the whole receiver must be enough to bring the output of the last IF tube to a value approaching saturation on noise alone. Screen dropping resistors may be used to safeguard the tubes under this condition.

IX. Display of Signals from a Logarithmic Receiver

When the output from a logarithmic amplifier is to be fed to a PPI display, special consideration must be given to the design of the video amplifier preceding the cathode-ray tube. Normally such amplifiers are adjusted so that full painting on the tube is obtained for echoes of power 10 db above noise. With a linear receiver such an echo will give an output amplitude signal-to-noise ratio of about 3 times, but with a logarithmic receiver it will be much less. For instance, in one logarithmic receiver used by the author such a signal appeared at the output at about 50 per cent above noise level. The video amplification and display parameters must be adjusted to suit these new conditions.

X. The Bandwidth of Logarithmic Amplifiers

Figs. 10, 11, and 12 show response curves for the three amplifiers. These were obtained by varying the input level and frequency of input voltage to maintain the output constant at a level of 40 db below the saturation output of the receiver, that is, with the amplifier working at approximately the middle of its logarithmic range.

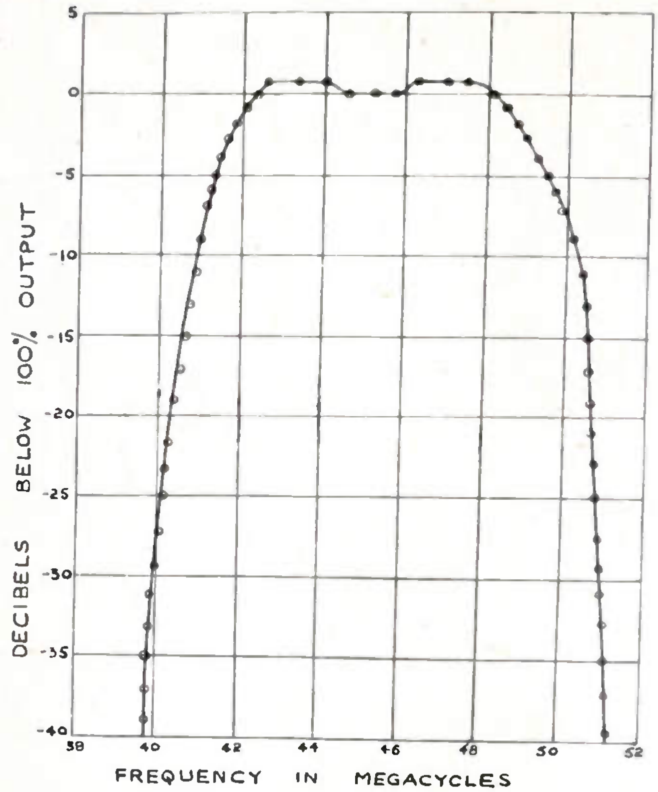

Fig. 10—Response curve for log receiver of 8-mc bandwidth, taken with a constant output 40 db below the saturation output of the receiver.

It should be noted that with logarithmic amplifiers the bandwidth is dependent upon the strength of signal applied. For weak signals the damping on each circuit is provided by the resistance across it together with the loading from the tube input impedance. The circuits will be adjusted for critical coupling under these conditions. Now as the input to the amplifier increases, the grid circuits of the band-pass pairs become one-by-one more heavily damped due to grid current than under the weak signal condition. Measurements on the amplifiers of 8- and 4-mc bandwidth showed the damping to increase by some 20 per cent through this effect. As a result the coupling drops below critical, and the bandwidth for strong signals, therefore, tends to be reduced.

Another consideration is that strong signals at the output of the amplifier will be built up by contributions from every detector stage, and each contribution will have passed through a different IF bandwidth gate.

The situation regarding bandwidth of logarithmic amplifiers is, therefore, a little obscure, and possibly some new definition of bandwidth is required in relation to them. However, several of the amplifiers described in this paper have been in continuous use on radar equipments, and when compared with linear amplifiers

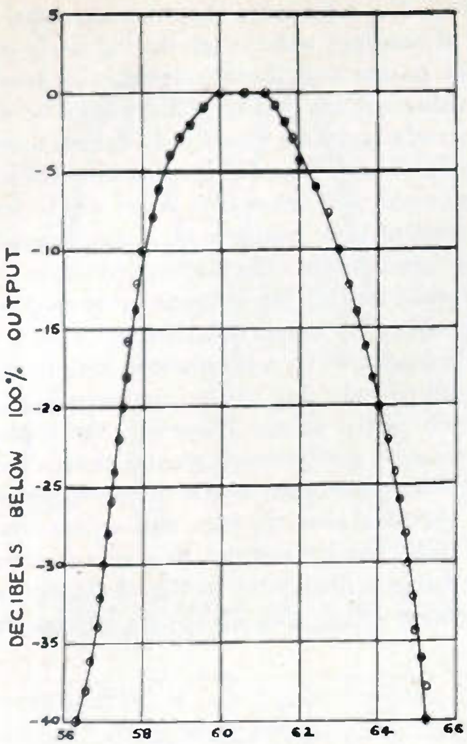

Fig. 11—Response curve for log receiver of 4-mc bandwidth, taken with a constant output 40 db below the saturation output of the receiver.

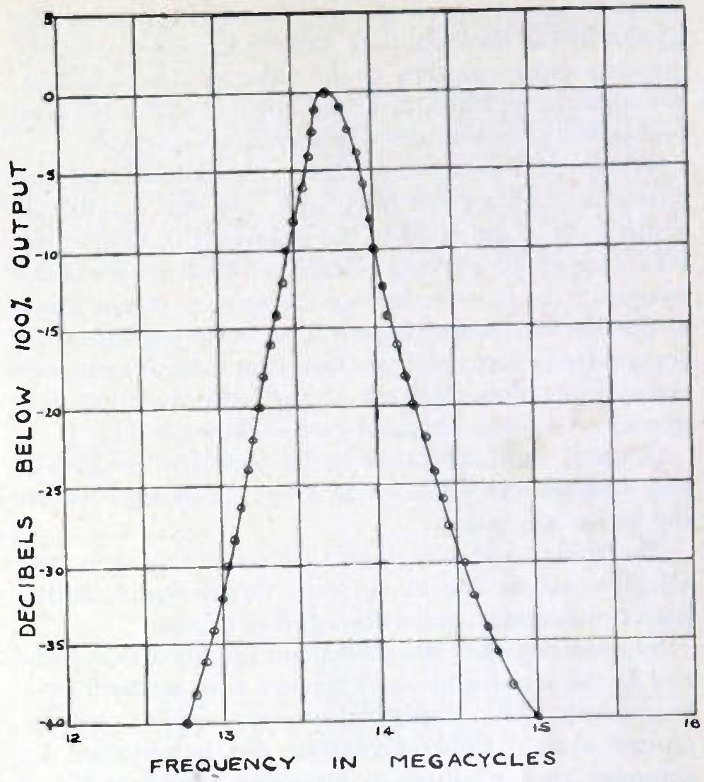

Fig. 12—Response curve for log receiver of 0.5-mc bandwidth, taken with a constant output 40 db below the saturation output of the receiver.

of similar bandwidth, no noticeable deterioration of pulse shape has been observed.

XI. Acknowledgments

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Resolution in Radar Systems*

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Summary—**The problem of azimuthal resolution in radar systems is discussed. It is shown how the resolution is related to the antenna beamwidth and thus in turn to the size of aperture. The possibilities of improving resolution beyond the normal capabilities is considered and the limitations are presented from two aspects. The fi st aspect is concerned with the attempt to build a "super-gain" antenna. The second aspect is concerned with the attempt to achieve a similar result through the utilization of filter equalization techniques. It is shown that no essential improvement in resolutlon appears to be obtainable in a practical manner.**

I. Introduction

RESOLUTION in a radar system is defined as the ability to distinguish between closely spaced objects. In the ordinary radar system, the ability to separate and distinguish between objects, or targets as they are called, is provided by two different techniques. The first is dependent on the pulse width of the system and provides resolution in range. This type of resolution will not be treated here. The second is dependent on the radiation field pattern and provides angular resolution either in azimuth, elevation, or both. This form of resolving power will be discussed in this paper.

In order to simplify the considerations, resolution in one plane only will be covered, namely, resolution in the azimuthal plane. Such resolution is normally provided by a fan beam having a narrow beam angle in the horizontal plane and rotating at a uniform angular velocity about an axis perpendicular to the horizontal plane. The analysis and conclusions may be applied to elevation angular coverage and also to the combination of both.

The field-amplitude pattern of an antenna with a uniformly illuminated aperture is a $(\sin x)/x$ function. The scale is related to the aperture. In the normal radar system, the same antenna is used for transmitting and receiving. The received signal field will then be propor-

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